SEMI-CLASSICAL QED AKA, NRQED NRQM

CLASSICAL EM: $\vec{E}, \vec{B}, \vec{A}, \varphi$

NR QM: SCHRODINGER FORMULATION H

FEYNMAN FORMULATION L

FIRST QUANTIZATION: MATTER IS QUANTIZED

SECOND QUANTIZATION: EM $\rightarrow$ PHOTONS

ACTUALLY, RQM $\vec{A} \rightarrow$ PHOTONS

H FOR HYDROGEN

$$H = \frac{p^2}{2m} + V(r) = \frac{p^2}{2m} + e\,\varphi(r)$$

EM SCALAR POTENTIAL

$$\varphi(\vec{r}) = \left|\psi(\vec{r})\right|^2$$

WHAT ABOUT $\vec{E}, \vec{B}, \vec{A}$?

CANONICAL MOMENTUM

$$\vec{\pi} = \vec{p} - \frac{e}{c}\vec{A}$$

$$H = \frac{\vec{\pi}^2}{2m} + e\,\varphi(\vec{r})$$

GAUGE INVARIANT ANGULAR MOMENTUM

$$\vec{L} = \vec{r} \times \vec{\pi}$$

H in terms of the potentials

$$H = \frac{1}{2m}\left(\vec{p} - \frac{e}{c}\vec{A}\right)^2 + e\,\varphi(\vec{r})$$

$$H = \frac{\vec{p}^{\,2}}{2m} - \frac{e}{c}\,\vec{p}\cdot\vec{A} - \frac{e}{c}\,\vec{A}\cdot\vec{p} + e\,\varphi(\vec{r}) + \frac{e^2}{c^2}\,\vec{A}^2$$

KE

PE

$H = H_0 + H_{int}$

H in terms of the fields

CLASSICALLY $\quad U = -\vec{p}\cdot\vec{E}$ electric dipole moment

$\quad U = -\vec{\mu}\cdot\vec{B}$ magnetic dipole moment

$$H_e = -\vec{p}\cdot\vec{E}$$

$$H_m = -\vec{\mu}\cdot\vec{B}$$

$\vec{L}$ => current => magnetic moment $\quad \vec{\mu}_L = \left(\frac{e}{2m}\right)\vec{L}$

$$= \gamma\vec{L}$$

$\vec{S}$ => magnetic moment $\quad \vec{\mu}_S \cong 2\left(\frac{e}{2m}\right)\vec{S}$

$$= g_e\,\gamma\,\vec{S}$$

$$H_m = -g_e\,\gamma\,\vec{S}\cdot\vec{B}$$

How many Nobel Prizes associated with spin?

Go to www.nobel.se Search for "spin" find 20 pages of results!!!

Even more directly, I count 13:

Physics 1902
"in recognition of the extraordinary service they rendered by their researches into the influence of magnetism upon radiation phenomena"
Hendrik Antoon Lorentz
Pieter Zeeman

Physics 1933
"for the discovery of new productive forms of atomic theory"

Erwin Schrödinger
Paul Adrien Maurice Dirac

Physics 1943
"for his contribution to the development of the molecular ray method and his discovery of the magnetic moment of the proton"
Otto Stern

Physics 1944
Isidor Isaac Rabi
"for his resonance method for recording the magnetic properties of atomic nuclei"

Physics 1945
PAULI, WOLFGANG
"for the discovery of the Exclusion Principle, also called the Pauli Principle"

Physics 1952
BLOCH, FELIX
PURCELL, EDWARD
"for their development of new methods for nuclear magnetic precision measurements and discoveries in connection therewith"

Physics 1994
Bertram N. Brockhouse

"for pioneering contributions to the development of neutron scattering techniques for studies of condensed matter"

"for the development of neutron spectroscopy"

Physics 1955
Polykarp Kusch
"for his precision determination of the magnetic moment of the electron"

Physics 1989

Norman F. Ramsey
"for the invention of the separated oscillatory fields method and its use in the hydrogen maser and other atomic clocks"

Hans G. Dehmelt
Wolfgang Paul
"for the development of the ion trap technique"

Physics 2001
Eric A. Cornell
Wolfgang Ketterle
Carl E. Wieman
"for the achievement of Bose-Einstein condensation in dilute gases of alkali atoms, and for early fundamental studies of the properties of the condensates"

Chemistry 1991
ERNST, RICHARD R.
"for his contributions to the development of the methodology of high resolution nuclear magnetic resonance (NMR) spectroscopy"

Chemistry 2002
Kurt Wüthrich
"for the development of methods for identification and structure analyses of biological macromolecules"

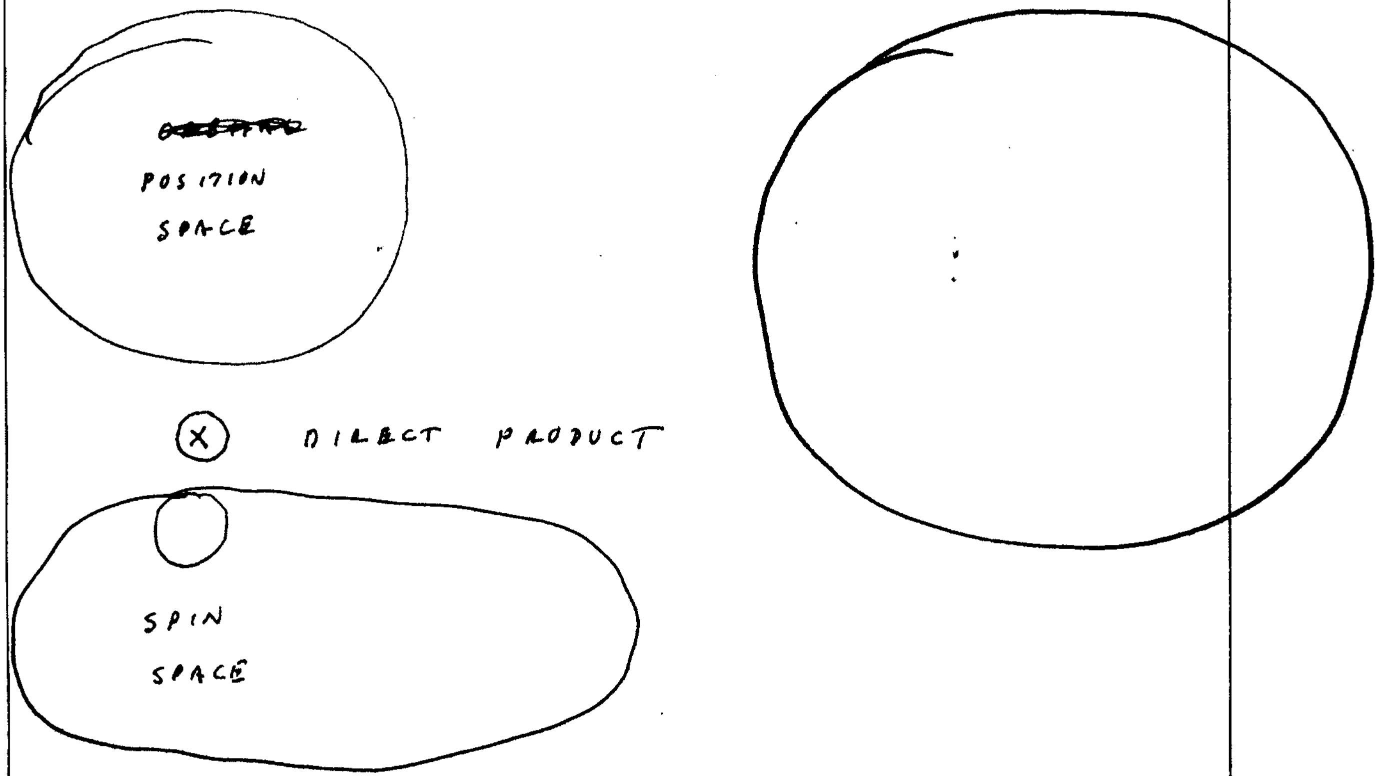
"for his development of nuclear magnetic resonance spectroscopy for determining the three-dimensional structure of biological macromolecules in solution"

Medicine 2003
"for their discoveries concerning magnetic resonance imaging"
Paul C. Lauterbur
Sir Peter Mansfield

TO INCLUDE THE EFFECTS OF SPIN,

WE NEED ADDITIONAL DEGREES OF FREEDOM

$\Longrightarrow$ MUST MAKE HILBERT SPACE "BIGGER"



- FOR SPIN 1/2

$$\begin{array}{c} \psi(\vec{r},t) \\ \otimes \\ \begin{pmatrix} a \\ b \end{pmatrix} \end{array} \longrightarrow \Psi(\vec{r},t) = \begin{pmatrix} \psi_+(\vec{r},t) \\ \psi_-(\vec{r},t) \end{pmatrix}$$

FOR SPIN 1

$$\left.\begin{matrix} \psi(\vec{r},t) \\ \otimes \\ \begin{pmatrix} a \\ b \\ c \end{pmatrix} \end{matrix}\right) \longrightarrow \quad \underline{\Psi}(\vec{r},t) = \begin{pmatrix} \psi_+(\vec{r},t) \\ \psi_0(\vec{r},t) \\ \psi_-(\vec{r},t) \end{pmatrix}$$

SPIN OPERATORS ACT ON SPIN ~~SPACE~~ PIECE

ORBITAL OPERATORS ACT ON ORBITAL PIECE

MIXED OPERATORS ACT ON BOTH PIECES

spin 1/2

SPIN OPERATORS

$$S_+ = \frac{1}{2}\left(S_x + i S_y\right) = \hbar \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$S_- = \frac{1}{2}\left(S_x - i S_y\right) = \hbar \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$S_+ \, \underline{\Psi}(\vec{r}) = \hbar \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \psi_+(\vec{r}) \\ \psi_-(\vec{r}) \end{pmatrix} = \hbar \begin{pmatrix} \psi_-(\vec{r}) \\ 0 \end{pmatrix}$$

$$S_- \, \underline{\Psi}(\vec{r}) = \hbar \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \psi_+(\vec{r}) \\ \psi_-(\vec{r}) \end{pmatrix} = \hbar \begin{pmatrix} 0 \\ \psi_+(\vec{r}) \end{pmatrix}$$

WHAT IS SPIN?

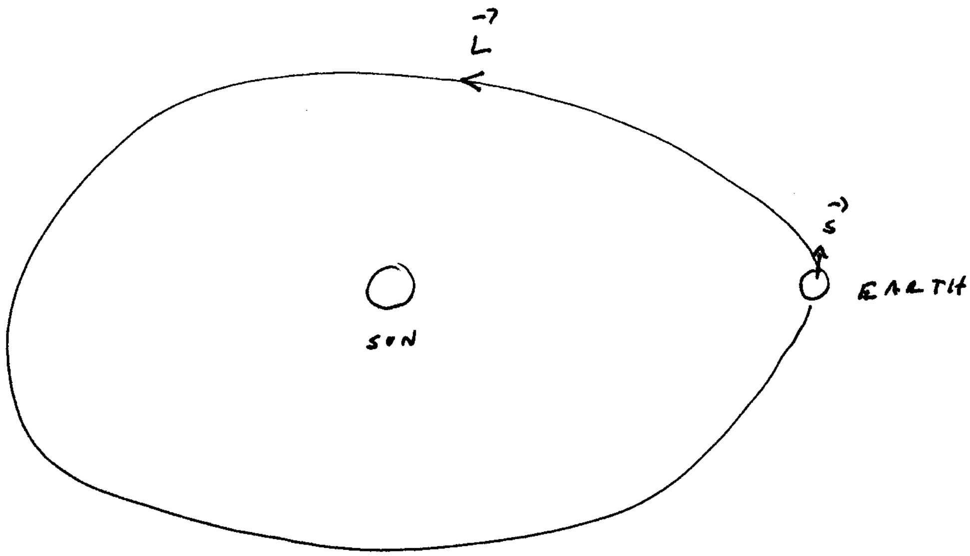
REAL PARTICLES HAVE INTRINSIC ANGULAR MOMENTUM.

THE ASSOCIATED DEGREE OF FREEDOM IS CALLED SPIN ANGULAR MOMENTUM $\Rightarrow$ SPIN

FIXED PROPERTY OF THE PARTICLE

CANNOT CHANGE IT! YOU CAN CHANGE $\vec{L}$.

CLASSICAL PICTURE



OKAY, MAYBE THE ELECTRON IS LIKE A LITTLE SPINNING BALL.... HOW FAST DOES IT SPIN?

mass $\Rightarrow$ $\vec{S}$

charge $\Rightarrow$ $\vec{\mu}$

3 - The classical electron radius

Another characteristic length scale is the length scale at which renormalization becomes really important. Renormalization is an aspect of field theory which deals with such issues as the fact that the electromagnetic 'd produced by an electron has energy and thus should be counted as part of the mass of the electron! The .gth scale at which these effects become really important is called the classical electron radius. It's important to note that it is really *classical*, not quantum mechanical, because it only depends on classical electromagneticsm, which doesn't involve hbar, and the formula for the rest energy of an electron, which involves c but not hbar. Indeed, renormalization was an issue in classical field theory before quantum field theory came along.

So the classical electron radius should just depend on the mass of the electron, its charge, and the speed of light. Recall these have units

- m = M
- e = $L^{3/2}M^{1/2}/T$
- c = L/T

so to get a length out of these we should form e^2/mc^2. So, without doing any real work, we can guess

$r_e = e^2/mc^2$.

We can derive the classical electron radius by working out the electric field outside of a ball having charge equal to that of the electron, e, and radius L, then working out the energy of this electric field, and then setting that energy equal to the electron mass m. Solving for L we get a formula for the electron radius r_e. In other words, the classical electron radius is the radius the electron would have to have for all of its mass to be due to the electric field it produced, assuming it was a charged shell. Up to miscellaneous factors we get

$r_e = e^2/mc^2$,   HOW BIG IS AN ELECTRON?

of course; since the actual calculation is not very exciting I'll skip it.

.. s worth noting that the classical electron radius is 1/137 as big as the Compton wavelength of the electron - the all-important fine structure constant again! So we have 3 length scales:

- Bohr radius r - about 5×10^{-11} meters
- Compton wavelength $L_{Compton}$ - about 4×10^{-13} meters
- Classical electron radius r_e - about 3×10^{-15} meters

each of which is 1/137 as big as the previous one. The Bohr radius depends only on hbar, e, and m. The Compton wavelength depends only on hbar, c, and m. The classical electron radius depends only on e, c, and m. Nice set-up, huh? I suppose I should relent and tell you that this mysterious number 1/137, the fine structure constant, is just   FINE STRUCTURE CONSTANT

e^2/hbar c. $\alpha = \frac{e^2}{\hbar c} = \frac{1}{137}.$

It's a dimensionless constant depending only on hbar, e, and c. In this respect it's more fundamental than any of the length scales mentioned, because all the length scales mentioned involve the electron mass, and one could work them out for particles other than the electron, whereas

e^2/hbar c

is truly universal, once you remember that the "electron charge" is nothing specific to the electron but is a basic aspect of electromagnetism that applies to all charged particles. (Yes, quarks apparently have charge 1/3, but that doesn't really affect my point.) In other words, the fine structure constant is a dimensionless measure of how strong the electromagnetic force is, and we have seen that it sets the ratio of 3 important length scales.

· The Planck length

Now for one final length scale - still smaller. This is the length scale at which quantum gravity should become important - the Planck length l. On the scale of the Planck length, it's possible that the structure of

HOW BIG IS AN ELECTRON?

TWO CLASSICAL ANSWERS

(1) CLASSICAL RADIUS r_e

MNEMONIC NOT PHYSICS

$a_0 = 0.529177 \; 10^{-10} m$

COULOMB ENERGY = REST MASS ENERGY

$$\frac{e^2}{r_{e_0}} = m c^2$$

$$r_{e_0} = \frac{e^2}{m c^2} = 2.8179 \times 10^{-15} m$$

$$\sim 3 \times 10^{-5} \text{ Å} = \frac{a_0}{(137)^2}$$

much smaller than an atom

PHYSICAL MEANING: EM cross section

(2) COMPTON RADIUS

$$= 3.8616 \times 10^{-13} m$$

$$r_c = \frac{\hbar}{mc} \sim 4 \times 10^{-3} \text{ Å}$$

$$r_c = \frac{a_0}{137} \qquad \left(\frac{\hbar}{mc}\right) \frac{c}{c} \frac{e^2}{e^2} = \left(\frac{\hbar c}{e^2}\right)\left(\frac{e^2}{m_0 c^2}\right)$$

PHYSICAL MEANING:

$$= \frac{r_0}{\alpha} \approx 137 \, r_0$$

THOMPSON SCATTERING

LOW ENERGY ELASTIC SCATTERING

$$\vec{E} = E_0 \hat{x} \cos(kz - \omega t)$$

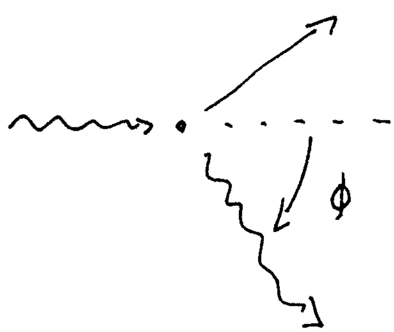
$$\frac{d\sigma}{d\Omega} = r_e^2 \, |\epsilon_i \cdot \epsilon_s|^2$$

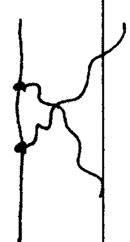
$$= r_e^2 \sin^2\theta .$$

SEAGULL DIAGRAM

COMPTON SCATTERING

INELASTIC SCATTERING





$$\Delta\lambda = \frac{h}{mc}(1 - \cos\phi) = 0.02426\,\text{Å}\,(1 - \cos\phi)$$

$$\frac{d\sigma}{d\Omega} = r_c^2 = (137)^2 (r_0)^2$$

$$r_c = \frac{r_0}{\alpha} = 137\, r_0$$

12'

HOW FAST WOULD IT SPIN?

$$L = \frac{1}{2}\hbar$$

$$L = I\omega = \left(\frac{2}{5} m R^2\right)\left(\frac{v}{R}\right) = \frac{2}{5} m v R$$

$$\Rightarrow \quad v = \frac{5}{4} \frac{\hbar}{m R}$$

$$R_e = \frac{e^2}{m_e c^2}$$

$$\boxed{v_0 = 171\ c\ !}$$ SUPERLUMINAL!

$$R_C = \frac{1}{\alpha}(R_{e0})$$

$$R_C \approx 137\, r_0$$

$$\boxed{v_0 = \frac{171}{137} = 1.25\ c}$$ STILL SUPERLUMINAL...

13

PRESENT UNDERSTANDING: the electron is a point particle

$\Rightarrow$ NOTHING INSIDE THAT CAN SPIN!

$\boxed{\text{THE PARTICLE ZOO}}$

LEPTONS

all $s = 1/2$ all leptons are point particles

e	ν_e	e^+	$\bar{\nu}_e$	$\{u, d\}$
μ	ν_μ	μ^+	$\bar{\nu}_\mu$	$\{c, s\}$
τ	ν_τ	τ^+	$\bar{\nu}_\tau$	$\{t, b\}$

HADRONS all hadrons are composite particles

BARYONS: THREE BOUND QUARKS

spin $1/2$: $p, n, \Sigma^+, \Sigma^0, \Sigma^-, \Lambda^0, \Xi^0\ \Xi^-$

spin $3/2$: Ω^-

MESONS: TWO BOUND QUARKS

SPIN 0	π^+	K^+	η^+
	π^0	$K^0\quad \bar{K}_0$	η^-
	π^-	K^-	

FOUR FORCES

GRAVITY	GRAVITONS	SPIN 2
E & M	PHOTONS	SPIN 1
WEAK	GAUGE BOSONS Z_0 $W^{\pm}$	SPIN 1
STRONG	GLUONS	SPIN 1 8 COLORS

SPIN EFFECTS IN ATOMS

spin 1/2 proton

spin 1/2 electron

SPINS INTERACT => ENERGY LEVELS CHANGE

ADD

~~ELECTRONIC ENERGY SCALE~~ $\cancel{E_0 = -13.6\,eV}$

THE ENERGY SCALES:

ELECTRONIC $\quad \frac{e^2}{r} \qquad E_0 = 13.6 \text{ eV}$

SPIN - ORBIT INTERACTION

FINE STRUCTURE

$$H_{FS} = A \, \vec{L} \cdot \vec{S} \propto \alpha^2 E_0 \qquad \sim 10^{-2} \text{ eV}$$

SPIN - SPIN INTERACTION

HYPERFINE STRUCTURE

$$H_{HF} = A \, \vec{S}_1 \cdot \vec{S}_2 \propto \left(\frac{m}{M}\right) \alpha^2 E_0 \qquad \sim 10^{-7} \text{ eV}$$

FINE STRUCTURE CONSTANT

$$\alpha = \frac{e^2}{\hbar c} \sim 137.$$